

(A5) Let the two-digit number have its digits a and b respectively with a being the first digit and b being the second digit.

So our two-digit number is ' ab ' = $10a + b$, where $0 \leq b \leq 9$

Difference between the first & second digit = $|b - a|$ $\begin{cases} (a - b) \text{ if } a > b \\ (b - a) \text{ if } b > a \end{cases}$ and $1 \leq a \leq 9$. 2 cases.

We consider the 2 cases separately:

Case 1 ($a > b$)

$(a-b)$ divides $(10a+b)$ and $(a-b)$ so it divides their sum $= (10a+b) + (a-b) = 11a$.

That is, $(a-b)$ divides $11a$. $\begin{cases} (a-b) \mid 11 \\ (a-b) \mid 1 \end{cases} \Rightarrow (a,b) = (b+1, b)$ impossible

$(a-b)$ divides $a \rightarrow a = k(a-b) \Rightarrow kb = (k-1)a \rightarrow k \geq 2$

If $k=1 \rightarrow b=0 \rightarrow (a,b) = (a,0)$.

If $k \geq 2 \rightarrow$ since $\text{HCF}(k, k-1) = 1 \rightarrow k | a$ in (*) $\Rightarrow a = mk \rightarrow b = (k-1)m$

$\rightarrow a-b = m$; now $m=1$ (already considered in $a-b=1$). So $m \geq 2 \quad 2 \leq m < 5$

Also note that $10 > a = mk \geq 2m \rightarrow 2m < 10$ or $m < 5 \rightarrow m = 2, 3$ or 4 .

So, from the above, we check for: $a-b = 2, 3$ or 4 .

$a-b=1 \rightarrow$ yielding the possible solutions of the form: $(b+1, b) = (1,0) (2,1) (3,2) (4,3) (5,4) (6,5) (7,6) (8,7) (9,8)$

$a-b=2 \rightarrow$ yielding the possible solutions of the form: $(b+2, b) = (2,0) (4,2) (5,3) (6,4) (8,6)$

$a-b=3 \rightarrow$ yielding the possible solutions of the form: $(b+3, b) = (3,0) (6,3) (9,6)$

$a-b=4 \rightarrow$ yielding the possible solutions of the form: $(b+4, b) = (4,0) (8,4)$

and $b=0 \rightarrow$ yielding the possible solutions of the form: $(9,0) = (5,0) (7,0) (18,0) (9,0)$

Repeted values are omitted

There are 23 Solutions in Case 1.

Case 2 ($a < b$)

$b-a$ divides the difference: $(10a+b) - (b-a) = 11a$. So $(b-a)$ divides $11a \rightarrow b-a = 11$ (impossible)

Similarly, $b-a | a \rightarrow a = h(b-a) \rightarrow a(h+1) = hb \rightarrow h=1 \rightarrow 2a=b \rightarrow (a,b) = (1,2) (2,4) (3,6) \text{ or } (4,8) \rightarrow$ each occurs once $\rightarrow 2n < 10 \rightarrow n < 5 \rightarrow n = 1, 2, 3 \text{ or } 4 \rightarrow$ we check for correspondingly, $b-a = 1, 2, 3 \text{ or } 4$:

$b-a=1 \rightarrow$ Possible solutions: $(1,2) (2,3) (3,4) (4,5) (5,6) (6,7) (7,8) (8,9)$

$b-a=2 \rightarrow$ Possible solutions: $(2,4) (4,6) (6,8)$

$b-a=3 \rightarrow$ Possible solutions: $(3,6) (6,9)$

$b-a=4 \rightarrow$ Possible solutions: $(4,8)$

There are 14 Solutions in Case 2. \rightarrow Thus, total number of = 23 + 14 = 37

Answer: 37

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from Case 1
from Case 2

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