(B3) The key in this problem is to note the following algebraic identity:

$$\frac{\sqrt{x}-\sqrt{1007}}{x-1007}+\frac{\sqrt{2014-x}-\sqrt{1007}}{1007-x}=\frac{\sqrt{2014-x}-\sqrt{x}}{1007-x}$$

which holds true with:

(1) $\sqrt{2014-x}$ must be real in which case it must be a "square root of a ~~positive~~ ~~nonnegative~~ nonnegative integers" (because we are only interested in positive integer solutions for x): $2014-x \geq 0$ or $x \leq 2014$, x a positive integer

and

(2) Denominator of any fraction above $\neq 0$ because division by 0 is not defined. So $x-1007 \neq 0$ or $x \neq 1007$

Combining (1) and (2) The identity holds when:

(i) x is a positive integer

(ii) $x \leq 2014$

(iii) $x \neq 1007$

That is, when

$x = 1, 2, 3 \ldots 1006, 1008, \ldots 2014$, (excluding 1007 from 1 to 2014).

Thus we have found (2013) solutions which satisfy the algebraic identity above.

Next, we show that the algebraic identity above is equivalent to the equation for our question ($x \neq 1007$). For this note that (we begin with our equation):

All these are equal to 1

(1) $$\frac{1}{\sqrt{x}+\sqrt{1007}}=\frac{1}{\sqrt{x}+\sqrt{1007}}\times\frac{\sqrt{x}-\sqrt{1007}}{\sqrt{x}-\sqrt{1007}}=\frac{\sqrt{x}-\sqrt{1007}}{x-1007}$$

(2) $$\frac{1}{\sqrt{2014-x}+\sqrt{1007}}=\frac{1}{\sqrt{2014-x}+\sqrt{1007}}\times\frac{\sqrt{2014-x}-\sqrt{1007}}{\sqrt{2014-x}-\sqrt{1007}}=\frac{\sqrt{2014-x}-\sqrt{1007}}{1007-x}$$

(3) $$\frac{2}{\sqrt{x}+\sqrt{2014-x}}=\frac{2}{\sqrt{x}+\sqrt{2014-x}}\times\frac{\sqrt{2014-x}-\sqrt{x}}{\sqrt{2014-x}-\sqrt{x}}=\frac{\sqrt{2014-x}-\sqrt{x}}{1007-x}$$

(1)+(2), Adding First two of the above and writing only the leftmost and rightmost terms as our LHS and RHS respectively, we obtain:

$$\frac{1}{\sqrt{x}+\sqrt{1007}}+\frac{1}{\sqrt{2014-x}+\sqrt{1007}}=\frac{\sqrt{x}-\sqrt{1007}}{x-1007}+\frac{\sqrt{2014-x}-\sqrt{1007}}{1007-x}$$

Noting that our equation in the question requires the LHS = (3) → at Leftmost and similarly the RHS of our last equation is such that RHS = (3) → at Rightmost (because of the algebraic identity at the start).

We have, our equation = algebraic identity (for all x, except $x \neq 1007$)

This means our equation holds true or is satisfied by the 2013 integers (1 to 1006 and 1008 to 2014)

Now our equation does not have the restriction $x \neq 1007$ simply because its denominators do not become 0 when $x = 1007$. So we check for this particular value (by putting $x = 1007$ in our equation) →

$$\frac{1}{\sqrt{1007}+\sqrt{1007}}+\frac{1}{\sqrt{2014-1007}+\sqrt{1007}}=\frac{2}{\sqrt{1007}+\sqrt{2014-1007}}$$

Thus $x = 1007$ also satisfies our equation.

Pg 15 We have found our 2014 positive integer solutions as → Answer: All integers from 1 to 2014