

A3

Firstly note the factorization:

$$\begin{aligned}
n^3 - n^2 - n + 1 &= n^2(n-1) - (n-1) \\
&= (n^2 - 1)(n-1) \\
&= (n+1)(n-1)(n-1) \\
&= (n+1)(n-1)^2
\end{aligned}$$

Since  $n$  is an integer and we're told to require  $(n^3 - n^2 - n + 1)$  to be a perfect square, (i.e, the LHS of the above is a perfect square), so it must be that " $(n+1)$  too is a perfect square" (because the RHS being a perfect square must be a product of two perfect squares since we're already given or can see that  $(n-1)^2$  is a perfect square).

We're also given in the problem that  $n$  lies between 200 and 250. This means that we need to find a perfect square called  $(n+1)$  such that  $201 < (n+1) < 251$ .

Since the only perfect square available\* is  $225^*$ , thus  $(n+1) = 225$ . or equivalently  $n = 225 - 1$   
Or simply

$$n = 224$$

\* Note that 225 is the only perfect square which lies between 201 and 251 because

Consecutive Squares

$$\begin{cases}
14^2 = 196 < 201 \\
15^2 = 225 \\
16^2 = 256 > 251
\end{cases}$$

Answer: 224  
for  $n$