A4 For any three sets of points forming a triangle with area $\frac{1}{2}$, we label the coordinates ~~was~~ as follows: coordinate (x, y) Note that: $1 \leq x \leq 2$, $1 \leq y \leq 10$

1. Let the coordinates be: (x, y) (a, b) and (c, d).
2. Let x be labelled such that x is the smallest x-coordinate: as follows: ~~$x \leq a$ and $x \leq c$~~ $x \leq a \leq c$. (So our pairs are ordered).

We do the labelling such to ease computation by breakdown into cases:

because $1 \leq x \leq 2$: Case 1 → $x = 1$; Case 2 → $x = 2$

$$\text{Area of triangle} = \frac{1}{2} \Rightarrow \begin{vmatrix} x & a & c & x \\ y & b & d & y \end{vmatrix} = \frac{1}{2}$$

reduces to → $bx + ad + cy - ay - bc - dx = \pm 1$

→ Since the ± 1 sign for area is due to some orientation (clockwise/anticlockwise) ~~of~~ location of the points, we may take the area to be $+1$ without loss of generality (WLOG) since it's still made up of the same set of three coordinates.

→ So from henceforth, we take

$$bx + ad + cy - ay - bc - dx = 1 \quad (1)$$

Consider each case separately:

Case 1 $[x = 1]$

Equation 1 becomes:

$$b + ad + cy = 1 + bc + ay + d$$

Yield ~~two~~ Three possible solution's only for (a, c).

[note $(a, c) = (2, 1)$ cannot since $a \leq c$].

First possibility → $(a, c) = (1, 1)$ → $b + d + y = 1 + b + y + d$ → $1 = 0$ (impossible)

Second possibility → $(a, c) = (1, 2)$ → $b + d + 2y = 1 + 2b + y + d$ → $y = 1 + b$ → 9 solutions for $(y, b) =$ $(10, 9)$ $(9, 8)$ $(8, 7)$ $(7, 6)$ $(6, 5)$ $(5, 4)$ $(4, 3)$ $(3, 2)$ $(2, 1)$

and for any value of d. There are 10 values for d. Thus there are a Total of $9 \times 10 = 90$ such triangles.

Third possibility → $(a, c) = (2, 2)$

→ $b + 2d + 2y = 1 + 2b + 2y + d$

→ $d = 1 + b$ → 9 solutions for $(d, b) = (10, 9)$ $(9, 8)$ $(8, 7)$ $(7, 6)$ $(6, 5)$ $(5, 4)$ $(4, 3)$ $(3, 2)$ $(2, 1)$ and

Again, for any value of y (from 1 to 10) ~~the~~ each of the above produces 10 new solutions giving a total of $9 \times 10 = 90$ more triangles.

Case 2 $[x = 2]$

So since by ordering → $2 = x \leq a \leq c \leq 2$ → it must be that $a = c = 2$.

Hence we only need to consider the possibility that $(a, c) = (2, 2)$.

Equation 1 becomes, $2b + 2d + 2y = 2y + 2b + 2d + 1$ → $0 = 1$ (impossible)

Hence there are no solutions from this case.

Finally, the Total number of solutions is → 90 such triangles + 90 more triangles = 180 triangles in Total.

pg11

Answer: 180 triangles