

(A5) Consider the computation where  $\equiv_{10}$  denotes "has last digit of"  
 Given  $a_1 = 1$  and  $a_2 = 3$ . and  $a_n = a_{n-1} \times a_{n-2}$  for all  $n \geq 3$ .

$$\begin{aligned}
 a_1 &\equiv_{10} 2 \\
 a_2 &\equiv_{10} 3 \\
 a_3 &\equiv_{10} a_2 \times a_1 \equiv_{10} 3 \times 2 \equiv_{10} 6 \\
 a_4 &\equiv_{10} a_3 \times a_2 \equiv_{10} 6 \times 3 \equiv_{10} 8 \\
 a_5 &\equiv_{10} a_4 \times a_3 \equiv_{10} 8 \times 6 \equiv_{10} 8 \\
 a_6 &\equiv_{10} a_5 \times a_4 \equiv_{10} 8 \times 8 \equiv_{10} 4 \\
 a_7 &\equiv_{10} a_6 \times a_5 \equiv_{10} 4 \times 8 \equiv_{10} 2 \\
 a_8 &\equiv_{10} a_7 \times a_6 \equiv_{10} 2 \times 4 \equiv_{10} 8 \\
 a_9 &\equiv_{10} a_8 \times a_7 \equiv_{10} 8 \times 2 \equiv_{10} 6 \\
 a_{10} &\equiv_{10} a_9 \times a_8 \equiv_{10} 6 \times 8 \equiv_{10} 8 \\
 a_{11} &\equiv_{10} a_{10} \times a_9 \equiv_{10} 8 \times 6 \equiv_{10} 8 \\
 a_{12} &\equiv_{10} a_{11} \times a_{10} \equiv_{10} 8 \times 8 \equiv_{10} 4 \\
 a_{13} &\equiv_{10} a_{12} \times a_{11} \equiv_{10} 4 \times 8 \equiv_{10} 2 \\
 a_{14} &\equiv_{10} a_{13} \times a_{12} \equiv_{10} 2 \times 4 \equiv_{10} 8 \\
 a_{15} &\equiv_{10} a_{14} \times a_{13} \equiv_{10} 8 \times 2 \equiv_{10} 6 \\
 a_{16} &\equiv_{10} a_{15} \times a_{14} \equiv_{10} 6 \times 8 \equiv_{10} 8
 \end{aligned}$$

for last digit:  
 cycle: 6, 8, 8, 4, 2, 8  
 period 6.

for last digit:  
 cycle repeats: 6, 8, 8, 4, 2, 8  
 period 6.

Since the period for the repeating cycle of last digits is 6, we find the pattern in modulo 6 as follows:

$$\begin{aligned}
 a_{6k} &\equiv_{10} 4 \\
 a_{6k+1} &\equiv_{10} 2 \\
 a_{6k+2} &\equiv_{10} 8 \\
 a_{6k+3} &\equiv_{10} 6 \\
 a_{6k+4} &\equiv_{10} 8 \\
 a_{6k+5} &\equiv_{10} 8
 \end{aligned}$$

for any  $k \geq 0$  and  $k$  is a positive integer.

Returning to our problem,  
 (In particular) we find:  
 Since  $2014 = 6(335) + 4$   
 Hence it follows that:  $(k = 335)$   
 $a_{2014} \equiv_{10} a_{6(335)+4} \equiv_{10} 8$

Answer = 8