

A6 We proceed as follows:

$$\begin{aligned}
20! - 19! + 18! &= (20 \times 19 \times 18!) - (19 \times 18!) + 18! \\
&= 18! (20 \times 19 - 19 + 1) \\
&= 18! (19^2 + 1) \\
&= 18! (361 + 1) \\
&= 18! (362) \\
&= 362 \times 18! \\
&= 2 \times 181 \times 18!
\end{aligned}$$

So, we need to find the largest prime factor in  $(2 \times 181 \times 18!)$ . To begin with, we note that 2 is the smallest prime while the factor  $18!$  has its largest prime as 17 (because 17 is the largest prime less than 18). Hence we need to check whether 181 is a prime or if it has a prime factor larger than 17.

We will now proceed to show that 181 is a prime and thus it becomes the answer to our problem. To do this, we use the following well known fact about primes:

"For any positive integer  $n$ , if none of the primes less than  $\sqrt{n}$  divides  $n$ , then  $n$  must be a prime."

In our case,  $n = 181$ . Note that  $\sqrt{n} \approx \sqrt{181} < \sqrt{196} = 14$ . That is,  $\sqrt{n} < 14$ . So we need only check whether primes less than 14 divide 181 or not. (Primes less than 14: 2, 3, 5, 7, 11, and 13).

It's easy to see that none of these primes divide 181 and hence it follows that 181 is a prime. Thus 181 is our answer.

$$181 \div 2 = 90 \frac{1}{2}$$

$$181 \div 11 = 16 \frac{5}{11}$$

$$181 \div 3 = 60 \frac{1}{3}$$

$$181 \div 13 = 13 \frac{12}{13}$$

$$181 \div 5 = 36 \frac{1}{5}$$

$$181 \div 7 = 25 \frac{6}{7}$$

Answer: 181