OMK 2014 – MUDA (3 July 2015)

(B2) If $A = 2^N$ has 100 digits with smallest N then (2^{N-1} must have 99 digits) ~ (1) ∴ (* Note that 2^{N-1} cannot have 100-digits since it would contradict the minimality of N).

Why? We show that 2^{N-1} cannot have 98 digits or less.

Consider, if 2^{N-1} has 98 digits or less, then

$$2^{N-1} < 10^{98} \rightarrow \text{(The smallest 99-digit number)}$$

$$\Rightarrow \quad 2 \cdot 2^{N-1} < 2 \cdot 10^{98}$$

so,

$$2^N < 10 \cdot 10^{98}$$

$$= 10^{99} \text{ (The smallest 100-digit number)}$$

i.e.

$$2^N < 10^{99} \text{ (The smallest 100-digit number)}$$

$\Rightarrow$ (2^N has less than 100 digits) contradicting the condition required in the problem. Hence, (1) is true.

Now (1) implies that

(has exactly 99-digits) $\rightarrow$ $2^{N-1} < 10^{99}$ (The smallest 100-digit number).

so,

$$2 \times 2^{N-1} < 2 \times 10^{99}$$

or

$$2^N < 2 \times 10^{99} \sim (2)$$

(2) implies that the (first digit of 2^N is 1) ~ (3)

because $2^N < 2 \times 10^{99}$ (which is the smallest 100-digit number that begins with digit 2).

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