OMK 2014 – Sulong (3 July 2015)

B2 Given $A = \{1, 2, 3, \ldots, 2014\}$

Define $B = \{3, 4, 5, \ldots, 2014\}$ → Set A excluding elements 1 and 2

Now,

Number of Good Subsets = Number of subsets of B such that its sum of its elements is divisible by 3

= No. of subsets of the set $\{0, 1, 2, \ldots, 2011\}$ whose sum of its elements is divisible by 3.

= (No. of subsets of $\{1, 2, 3, \ldots, 2011\}$ whose sum of its elements is divisible by 3) + (No. of subsets of $\{1, 2, 3, \ldots, 2011\}$ whose sum of its elements is divisible by 3 add one extra element 0 to each subset formed)

No 0: Counts all subsets matching the condition which do not have element 0 as part of any subset formed here

Includes counting null set here

bijection counting (equal)

With 0: Counts all subsets satisfying the required condition which has element 0 in every subset formed here.

Includes counting ~~set~~ one element subset $\{0\}$ here

$\{0\}$

Corresponds to element 3 in the original set

Now since, every subset With 0 as part of its element can be obtained by a unique subset with No 0, there exists a bijection between these two sets of subsets implying that these two sets have an equal amount of subsets. Thus

$$\begin{pmatrix}\text{Number}\\ \text{of good}\\ \text{subsets}\end{pmatrix} = 2 \times \begin{pmatrix}\text{No. of subsets of}\\ \{1, 2, 3, \ldots, 2011\}\\ \text{whose sum of its}\\ \text{elements is divisible}\\ \text{by } 3\end{pmatrix} \quad (1)$$

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