OMK 2014 – Suleng (3 July 2015)

B2 Based on ①, it suffices to count "number of subsets of $\{1,2,3,\dots,2011\}$ whose sum of its elements is divisible by 3.

To count this, consider the generating function:

$$f(x) = \prod_{k=1}^{2011} (1+x^k)$$

$$= (1+x^1)(1+x^2)(1+x^3)\cdots(1+x^{2011})$$

Since $1+2+3+\dots+2011 = \frac{2011 \times 2012}{2} = 2023066$

$$= \sum_{j=0}^{2023066} a_j x^j$$

$$= 1 + x + x^2 + 2x^3 + \dots + x^{2023066}.$$

(several terms shown with their coefficients)

Note that $2023065 = 3 \times 674355$

In the above,

note that the coefficient of x^k is a_k.

The required sum is $(a_0 + a_3 + a_6 + \dots + a_{2023065})$

Also, it includes counting the null-set, ϕ.

Next, we apply the 3^{rd}-root of unity filter, computing $f(\omega^m)$ for primitive 3^{rd} root of unity, ω and $m \in \{1,2,3\}$ or $m \in \{0,1,2\}$ as follows: (note that)

$\omega^3 = 1$ ②

$1 + \omega + \omega^2 = 0$ ③

and so, $1+\omega = -\omega^2$ (3a)

$1+\omega^2 = -\omega$ (3b)

$\omega + \omega^2 = -1$ (3c)

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