OMK 2014 - Sulong (3 July 2015)

(B2) Next, we compute:

$$f(\omega^2) = \prod_{k=1}^{2011} (1+\omega^{2k})$$

$2011 \times 2 = 4022$

$$= (1+\omega^2)(1+\omega^4)(1+\omega^6)(1+\omega^8) \times \dots \times (1+\omega^{4022})$$

$$= \left(\prod_{m=1}^{670} (1+\omega^{2(3m)})\right) \times \left(\prod_{n=0}^{670} (1+\omega^{2(3n+1)})\right) \times \left(\prod_{l=0}^{669} (1+\omega^{2(3l+2)})\right)$$

Since ω^{2k} becomes

* $\omega^{2(3h)} = \omega^{3(2h)} = 1^{2h} = 1$

and,

* $\omega^{2(3h+1)} = \omega^{3(2h)} \cdot \omega^2 = 1^{2h} \times \omega^2 = \omega^2$

and,

* $\omega^{2(3h+2)} = \omega^{3(2h)} \times \omega^4 = 1^{2h} \times \omega^3 \times \omega = 1 \times 1 \times \omega = \omega$

$$= \left(\prod_{m=1}^{670} (1+1)\right) \times \left(\prod_{n=0}^{670} (1+\omega^2)\right) \times \left(\prod_{l=0}^{669} (1+\omega)\right)$$

$$= \left(\prod_{m=1}^{670} 2\right) \times \left(\prod_{n=0}^{669} (1+\omega^2)(1+\omega)\right) \times (1+\omega^2)$$

$$= 2^{670} \times \left(\prod_{n=0}^{669} (-\omega \times (-\omega^2)\right) \times (1+\omega^2)$$ (using 3b) (using 3a)

$$= 2^{670} \times \left(\prod_{n=0}^{669} \omega^3\right) \times (-\omega)$$ (using 3b)

$$= 2^{670} \times \left(\prod_{n=0}^{669} 1\right) \times (-\omega)$$

$$= 2^{670} \times (-\omega)$$

$$= -2^{670}\omega$$

i.e.

$$f(\omega^2) = -2^{670}\omega \quad (6)$$

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