Sulong

(A2) □829783 is a multiple of 3 means:

Sum of digits of □829783 is also a multiple of 3.

So let □+8+2+9+7+8+3 = 3m, where m is a positive integer.

37 + □ = 3m ⇝ {a multiple of 3}

Since □ is a digit, and being first digit:

$0 < \square \leq 9$

Thus 'quickly' trying values, □ = 1, 2, 3 ... 9

we find that, the only values which yield a multiple of 3 for the number "37 + □" are:

□ = 2, 5, 8

3m = 37 + □ = 39, 42, 45

But we also require HCF(9999, 3m) = 3.

Since 9 divides 9999, amongst the possible numbers (39, 42, 45), for 3m, we must choose the ones not divisible by 9. Note that 9 divides 45. So we're left with only 39 and 42. Furthermore, since

$9999 = 3^2 \times 11 \times 101$ where 3, 11 & 101 are primes,

For the possible digits 2 and 5

because Check:

[2] 829783 = 11 × 257253 → Reject 2 (causes HCF ≥ 11 × 3 = 33)

[5] 829783 is divisible by 3 only but not by 9, 11 or 101.

Hence □ = 5

Answer: 5