Sulong

$6 = 1(5)+1$

note: $2014 = 402(5)+4$

$2011 = 402(5)+1$

Add yields an odd number for S

(A3) Let $S = 1^{1!} + 2^{2!} + 3^{3!} + \ldots + 2014^{2014!}$

1007 odd numbers are added

Fact (1): $\to$ S is odd.

Why? $\to$ Consider its odd addends: $1^{1!} + 3^{3!} + 5^{5!} + \ldots + 2013^{2013!} \to$ odd number
+
$\to$ consider its even addends: $2^{2!} + 4^{4!} + 6^{6!} + \ldots + 2014^{2014!} \to$ even number

We use $1 \equiv_5 1$, $2 \equiv_5 2$, $3 \equiv_5 -2$, $4 \equiv_5 -1$ and $5 \equiv_5 0$, to get.

$$S \equiv_5 1^{1!} + 2^{2!} + (-2)^{3!} + (-1)^{4!} + 0^{5!} + \ldots + (-1)^{2014!} \quad \text{—— (2)}$$

Observe also that:
Since $(-2)^4 \equiv_5 2^4 \equiv_5 16 \equiv_5 1 \xrightarrow{\text{Thus}} (-2)^{4k} \equiv_5 2^{4k} \equiv_5 1$ —— (3)

and $4!, 5!, 6! \ldots 2014!$ are all multiples of 4, —— (4)

note $1^{4k} \equiv_5 (-1)^{4k} \equiv_5 1$

Facts (3) and (4) imply in (2) that:

$$S \equiv_5 1^{1!} + 2^{2!} + (-2)^{3!} + (-1)^{4!} + \underbrace{(1+1+1+1) \times (402)}_{\text{sets of remainders modulo 5 counted for 6 to 2014}}$$

$$S \equiv_5 1 + 4 + 64 + 1 + (4 \times 402)$$

$$S \equiv_5 1678$$

$$S \equiv_5 3$$

$\to$ means that S leaves a remainder of 3 when divided by 5

$\to$ Thus exists a positive integer h such that:

$$S = 5h + 3 \quad \text{—— (5)}$$

Now fact (1) together with fact (5) imply that:
For S to be odd, it must be that h is even.
This means that exists a positive integer l such that:

$$h = 2l \quad \text{—— (6)}$$

Finally, put (6) into (5) to obtain:

$\to S = 5(2l) + 3$

or $S = 10l + 3$

$\to$ We can conclude that $S \equiv_{10} 3$ because $10l + 3 \equiv_{10} 3$

$\boxed{\text{Answer: } 3}$