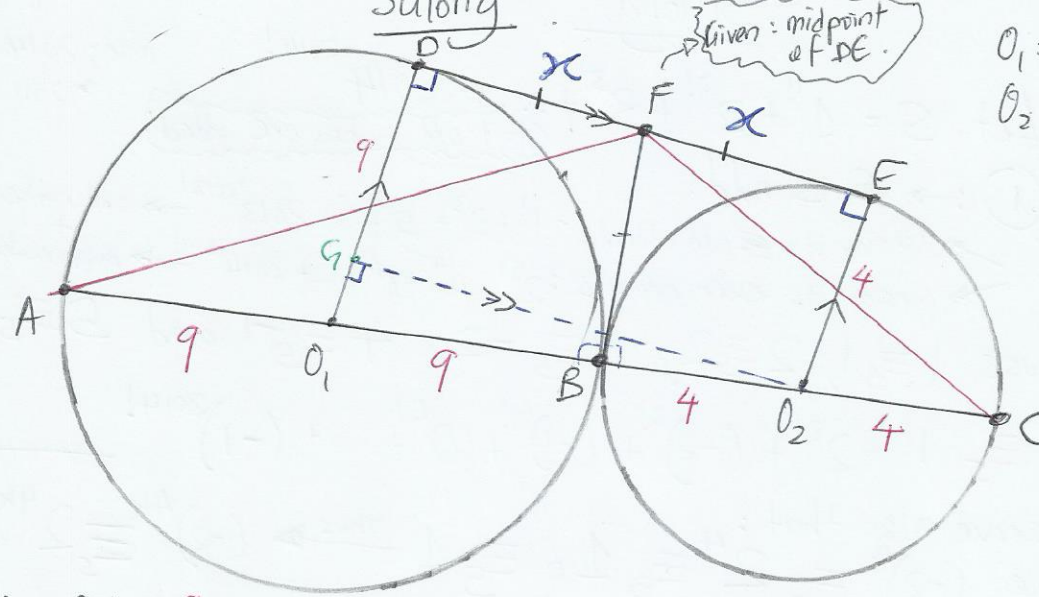
Solution

A4

O_1 = center of circle with diameter AB

O_2 = center of circle with diameter BC

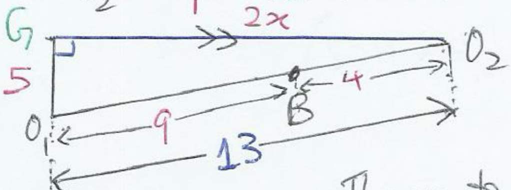


Note that $O_1D = 9$ being radius of circle with diameter AB and similarly, $O_2E = 4$ being radius of circle with diameter BC.

Also, $\angle O_1DF = \angle O_2EF = 90^\circ$ because DE is tangent to the circles, ~~respect~~ at D and E, respectively.

Next, let $DF = FE = x$.

To solve this problem, we first find the value of x. For this, draw line O_2G so that O_2G is parallel to ED. Next, consider the sides of right triangle, ΔO_1O_2G.



Note: $O_2G = ED = EF + FD = x + x = 2x$

and $O_1O_2 = O_1B + BO_2 = 9 + 4 = 13$.

Applying Phythagoras Theorem to this ~~tria~~ right triangle (ΔO_1O_2G),

$(2x)^2 = 13^2 - 5^2 \rightarrow 2x = 12 \rightarrow \boxed{x = 6}$ — (1)

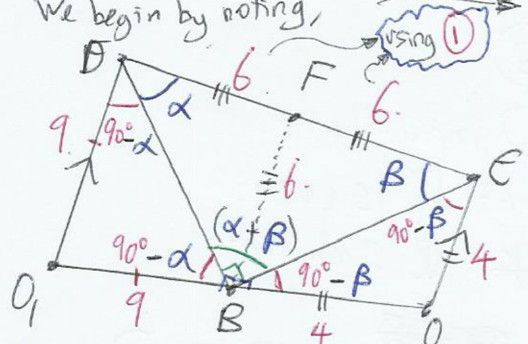
Next, we prove that $FB = x = 6$. How? Consider similar triangles ~~as follows~~..

We begin by noting,

Let $\angle BDE = \alpha$ and $\angle BED = \beta$.

Then $\angle O_1DB = 90^\circ - \alpha$ (because $\angle O_1DF = 90^\circ$, DE tangent at D).

Similarly $\angle O_2EB = 90^\circ - \beta$ (because $\angle O_2EF = 90^\circ$, DE tangent at E).



~~Since~~ This implies,

$\angle O_1BD = 90^\circ - \alpha$ (because $O_1D = O_1B$)

and $\angle O_2BE = 90^\circ - \beta$ (because $O_2B = O_2E$)

From these, we deduce, $\angle DBE = 180^\circ - \angle O_1BD - \angle O_2BE$ (angle in a straight line)

so, $\angle DBE = 180^\circ - (90^\circ - \alpha) - (90^\circ - \beta)$ or $\angle DBE = (\alpha + \beta)$

* Next, looking at the sum of angles in ΔBDE, we find that

$\alpha + \beta + (\alpha + \beta) = 180^\circ$

and hence, $\alpha + \beta = 90^\circ$, that is $\angle BDE = 90^\circ$ ~~...~~. Thus

angle in semicircle

$FB = FD = FE = 6$ because F is center of circle going through points D, B and E. and DE is diameter. $\boxed{FB = 6}$ — (2)

Finally, note that $AC = AB + BC = 18 + 8 = 26$,

That is, $\boxed{AC = 26}$ — (3)

Thus, Area of $\Delta AFC = \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times AC \times FB$

$= \frac{1}{2} \times 26 \times 6$

using (3)

using (2)

Note that: $FB = FE$ in ΔFBE implies $\angle FBE = \angle FEB = \beta$. Thus $\angle FBO_2 = \angle FBE + \angle EBO_2 = \beta + (90^\circ - \beta) = 90^\circ$

$\boxed{\text{Answer} = 78}$