

(A5) The idea is to convert  $12!$  seconds into years. Then, we would know Ara's age.

$$12! \text{ seconds} = \frac{12 \times 11! \times 5}{5} \text{ seconds}$$

$$= \frac{60 \times 11!}{5} \times \frac{1}{60} \text{ minutes}$$

$$= \frac{11!}{5} \text{ minutes}$$

$$= \frac{11! \times 60}{60 \times 5} \times \frac{1}{60} \text{ hour}$$

$$= \frac{11!}{2 \times 3 \times 5 \times 10} \text{ hour}$$

$$= \frac{11!}{2 \times 3 \times 5 \times 10} \times \frac{1}{24} \text{ day}$$

$$= \frac{11!}{2 \times 3 \times 5 \times 10 \times 4 \times 6} \text{ day}$$

$$= \frac{11!}{1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 10} \text{ day}$$

$$= \frac{11!}{6! \times 10} \text{ day}$$

$$= \frac{\cancel{11} \times \cancel{9} \times 11 \times \cancel{10} \times 9 \times 8 \times 7 \times 6!}{6! \times 10} \text{ day}$$

$$= 11 \times 9 \times 8 \times 7 \text{ day}$$

$$= 11 \times 9 \times 8 \times 7 \times \frac{1}{365} \text{ years}$$

$$= 11 \times 9 \times 8 \times 7 \times \frac{1}{5 \times 73} \text{ years}$$

$$= \frac{77}{5} \times \frac{72}{73} \text{ years}$$

$$= 15 \frac{2}{5} \times \frac{72}{73} \text{ years}$$

$$= 15.4 \times \frac{72}{73} \text{ years}$$

① and ② imply that  $\longrightarrow$

It follows that Ara is 15 years old.

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Note that:

$$15.4 \times \frac{72}{73} < 15.4 \times 1$$

$$\text{because } \frac{72}{73} < 1$$

$$\text{Since } 72 < 73$$

Thus, since

$$12! \text{ seconds} = 15.4 \times \frac{72}{73} \text{ years}$$

So,

$$12! \text{ seconds} < 15.4 \text{ years}$$

and

Note ~~that~~ also that:

$$11088 = 72 \times 154 > 73 \times 150$$

$$11088 = 72 \times 154 > 156 \times 73 = 10950$$

Imply that:

$$\frac{72}{73} > \frac{150}{154} = \frac{15.0}{15.4}$$

So,

$$12! \text{ seconds} = 15.4 \times \frac{72}{73} > 15 \text{ years}$$

i.e.  $12! \text{ seconds} > 15 \text{ years}$  ①

$$15 \text{ years} < \text{Ara's age (12! seconds)} < 15.4 \text{ years}$$

Answer: 15 years old.