

A6

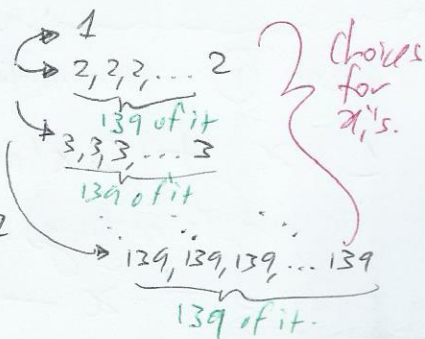
The idea to maximize the value of n is equivalent to using the most number of least scores (1 or 1 and 2 or 1 and 2 and 3 etc.) to satisfy $A_1 = 100$ and $A_2 = 139$ where A_i represents the score of the i -th place winner.

* Firstly, we note that if we use 1 only to form $A_1 = 100 \rightarrow n = 100 \rightarrow$ (A_2 formed using score 2 and above)
 $A_2 = 139 \rightarrow n = 139 \rightarrow$ (A_1 formed using score 2 and above)

Let's try each case to see if it's possible:

\rightarrow Since $A_2 = 139$ gives a larger possible value for $n = 139$, let's try out this possibility first. The question is: is it possible to form ~~with~~ the score $A_1 = 100$ using ~~exactly~~ exactly 139 numbers from the available: (One ~~1~~ 1, One hundred and thirty nine 2's, ... One hundred and thirty nine 139's).

Algebraically, does there exist integers, x_i 's such that: $A_1 = x_1 + x_2 + \dots + x_{139} = 100$ where x_i 's



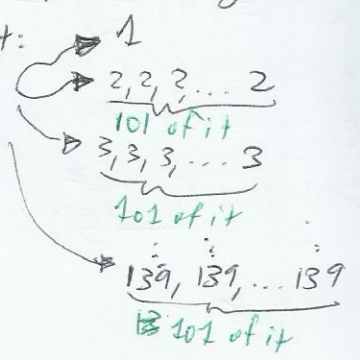
This is impossible since.

$100 = x_1 + x_2 + \dots + x_{139} \geq 1 + 2 \times (139 - 1) = 277$ yielding the contradiction $\rightarrow 100 \geq 277$.

\rightarrow Now we try out the other possibility in this scenario, namely:

$A_1 = 100$ formed using 1's only \rightarrow Since A_1 received $(n-1)$ scores of 1's, ~~thus~~ in this possibility, thus $(n-1) = 100 \rightarrow$ or equivalently $n = 101$. So as before, we have to try to form $A_2 = 139$ using exactly 100 positive integers as follows: Does there exist positive integers, y_i 's such that:

$A_2 = y_1 + y_2 + \dots + y_{100} = 139$ where choices for y_i 's



This is also impossible since,

$139 = y_1 + y_2 + \dots + y_{100} \geq 1 + 2 \times (100 - 1) = 199$ yielding the contradiction $\rightarrow 139 \geq 199$

So in the above, we have shown (proved) that using 1's only ~~to form~~ the scores $A_1 = 100$ or $A_2 = 139$ ~~leads to~~ leads to impossible scenarios (to form the other score, $A_2 = 139$ or $A_1 = 100$ respectively using the remaining available ranks).