(A6) The idea to maximize the value of n is equivalent to using the most number of least scores (1 or 1 and 2 or 1 and 2 and 3 etc..) to satisfy $A_1 = 100$ and $A_2 = 139$ where A_i represents the score of the i-th place winner.

* Firstly, we note that if we use 1 only to form $\rightarrow A_1 = 100 \rightarrow n = 101 \rightarrow$ (A_2 formed using score 2 and above)

or $A_2 = 139 \rightarrow n = \cancel{139}$ 140 $\rightarrow$ (A_1 formed using score 2 and above)

Let's try each case to see if it's possible:

→ Since $A_2 = 139$ gives a larger possible value for $n = \cancel{139}$ 140, let's try out this possibility first. The question is: is it possible to form ~~with~~ the score $A_1 = 100$ using exactly 139 numbers from the available: (One 1, One hundred and thirty nine 2's, ... One hundred and thirty nine 139's).

Algebraically, does there exist integers, x_i's such that:

$$A_1 = x_1 + x_2 + \dots + x_{139} = 100 \text{ where } x_i\text{'s}$$

Choices for x_i's: 1; $\underbrace{2,2,2,\dots 2}_{139 \text{ of it}}$; $\underbrace{3,3,3,\dots 3}_{139 \text{ of it}}$; ⋮ $\underbrace{139,139,139\dots 139}_{139 \text{ of it}}$

This is impossible since.

$$100 = x_1 + x_2 + \dots + x_{139} \geq 1 + 2\times(139 - 1) = 277$$

yielding the contradiction → $100 \geq 277$.

→ Now we try out the other possibility in this scenario, namely: $A_1 = 100$ formed using 1's only → Since A_1 received $(n-1)$ scores of 1's, in this possibility, thus $(n-1) = 100$ → or equivalently $n = 101$.. So as before, we have to try to form $A_2 = 139$ using exactly 100 positive integers as follows: Does there exist positive integers, y_i's such that:

$$A_2 = y_1 + y_2 + \dots + y_{100} = 139$$

the choices for y_i's: 1; $\underbrace{2,2,2,\dots 2}_{101 \text{ of it}}$; $\underbrace{3,3,3,\dots 3}_{101 \text{ of it}}$; ⋮ $\underbrace{139,139,\dots 139}_{101 \text{ of it}}$

This is also impossible since,

$$139 = y_1 + y_2 + \dots + y_{100} \geq 1 + 2\times(100-1) = 199.$$

yielding the contradiction → $139 \geq 199$

So in the above, we have shown (proved) that using 1's only ~~for~~ to form the scores $A_1 = 100$ or $A_2 = 139$ ~~yields~~ leads to impossible scenarios (to form the other score, $A_2 = 139$ or $A_1 = 100$ respectively using the remaining available ranks).