Sulong

(A6) → (Continued from the previous page)

Now, we're left to consider wether it's possible to form both A_1 and A_2 by using ~~scores~~ ranks 1 and 2 only. (which would imply the largest possible value of n) because using only 1's & 2's only would yield more integers in the formation of our rank-sum for both $A_1 = 100$ & $A_2 = 139$. → which in turn implies, (the more addends/ranks make up the rank-sum) implies → (the higher value of $(n-1)$ which is the number of addends/ranks. in the rank-sum)

So, this is equivalent to finding: ~~int~~ positive integers x and y such that:

(1) x ones are used in the rank-sum and $(n-1-x)$ two's are used in the rank-sum → $A_1 = 100$

(2) y ones are used and $(n-1-y)$ two's are used in the rank-sum → $A_2 = 139$

note that in both rank-sums, A_1 and A_2, we have exactly $(n-1)$ addends/ranks because

$x + (n-1-x) = (n-1)$

and $y + (n-1-y) = (n-1)$

each girl receives a score composed of $(n-1)$ other girl's ranks being summed up

Further, (1) written mathematically yields:

$$(1 \times x) + (2 \times (n-1-x)) = A_1 = 100$$

or just, (simplifying).

$$2n = 102 + x \quad \text{—(3)}$$

and similarly, (2) written mathematicaly yields:

$$(1 \times y) + (2 \times (n-1-y)) = A_2 = 139$$

Simplifying gives:

$$2n = 141 + y \quad \text{—(4)}$$

Note that since we want to use the most number of 1's, we try for this maximum possibility by considering

$$x + y = n \quad \text{—(5)}$$

because there are exactly n ranks of 1 available since each girl ranks 1 uniquely → no repeat

next step

Solve for possible values of $x, y,$ and n (Simultaneous equations in 3 unknowns):

Put (5) in (3) + (4), and solve for n:

$2n + 2n = 102 + 141 + (x+y)$

$4n = 243 + n$

$3n = 243 \rightarrow \boxed{n = 81}$

Put $n = 81$ back into (3) and (4) to get $x = 2(81) - 102 = 60$

$y = 2(81) - 141 = 21$

So, we're lucky since our best possibility so far works! yieding $n_{max} = 81$

Note:

$(x, y, n) = (60, 21, 81)$

Conclusion: To maximize n, we found that we can form the ~~f~~ given first place and second place scores using ranks 1 and 2 only (maximizing this usage of) as follows: (for $n = 81$)

$(1 \times 60) + (2 \times 20) = 100$ → First place's score

$(1 \times 21) + (2 \times 59) = 139$ → Second place's score.

$\boxed{\text{Answer: } n = 81}$