B1 $\Rightarrow$ (Continued from the previous page)

Multiplying out and simplifying:

$$(AB)h_1 + (MN)h_1 + (MN)h_2 + (DC)h_2 = (AB)h_1 + (AB)h_2 + (DC)h_1 + (DC)h_2$$

$$(MN)h_1 - (DC)h_1 = (AB)h_2 - (MN)h_2$$

$$h_1(MN - DC) = h_2(AB - MN)$$

$$\frac{(AB - MN)}{(MN - DC)} = \frac{h_1}{h_2} \quad \text{—} ②$$

Now, ① = ②, so $\dfrac{(DC+MN)}{(AB+MN)} = \dfrac{(AB-MN)}{(MN-DC)}$ both equal to $\dfrac{h_1}{h_2}$

Multiplying out and simplifying, we get:

$$MN^2 = \frac{AB^2 + DC^2}{2} \quad \text{—} ③$$

The key is to use the AM-GM inequality on the RHS of ③ as follows:

$$MN^2 = \frac{AB^2 + DC^2}{2} \geqslant \sqrt{AB^2 \cdot DC^2} = (AB)\cdot(DC)$$

That is, $MN^2 > AB \cdot DC$ — ④

We remove the equality possibility because the equality only happens if $AB = DC$ which is not the case in our problem since we're given that $DC > AB$

We now manipulate inequality ④ as follows:

$$MN^2 + DC\cdot MN > AB\cdot DC + DC\cdot MN$$ add $DC\cdot MN$ to both sides of the inequality

$$(DC+MN)\cdot MN > DC\cdot(AB+MN)$$

Dividing (by positive quantities) & rewriting,

$$\frac{(DC+MN)}{(AB+MN)}\cdot\frac{(MN)}{(DC)} > 1$$

using ①

$$\frac{h_1}{h_2}\cdot\frac{(MN)}{(DC)} > 1$$

continued here

equivalently (multiply up & down by $\frac{1}{2}$)

$$\frac{\frac{1}{2}\cdot h_1\,(MN)}{\frac{1}{2}\cdot h_2\cdot(DC)} > 1$$

Note that: $|\Delta BNM| = \frac{1}{2}h_1(MN)$ and $|NCD| = \frac{1}{2}h_2(DC)$

$$\frac{|\Delta BNM|}{|\Delta NCD|} > 1$$

or just, $|\Delta BNM| > |\Delta NCD|$

QED