Sulong.

(B3) (Continued from the earlier page)

We manipulate algebraically (7) as shown below:

$$\frac{y_i}{x_i} = (\log_2 y_i) \div (\log_2 x_i) \rightsquigarrow \text{(Rewriting)}$$

$$\frac{y_i}{x_i} = \left(\frac{\log_{x_i} y_i}{\log_{x_i} 2}\right) \div \left(\frac{\log_{x_i} x_i}{\log_{x_i} 2}\right) \rightsquigarrow \text{(Changing both logarithms to base } x_i\text{)}$$

$$\frac{y_i}{x_i} = \left(\frac{\log_{x_i} y_i}{\cancel{\log_{x_i} 2}}\right) \times \left(\frac{\cancel{\log_{x_i} 2}}{\log_{x_i} x_i}\right)$$

note $\log_{x_i} 2 \neq 0$

$$\frac{y_i}{x_i} = \frac{\log_{x_i} y_i}{\log_{x_i} x_i} \rightsquigarrow \text{(note that } \log_{x_i} x_i = 1\text{)}$$

$$\frac{y_i}{x_i} = \log_{x_i} y_i$$

$$x_i^{\frac{y_i}{x_i}} = y_i \rightsquigarrow \text{(by the definition of a logarithm)}$$

$$x_i^{y_i} = y_i^{x_i} \rightsquigarrow$$

Raise both sides to the x_i-th power

Thus we can easily see that by setting $x = x_i$ and $y = y_i$, we have infinitely many solutions in real numbers which satisfy the given equation in the problem, namely $x^y = y^x$.

Note that our x_i's and y_i's are infinitel since we defined $x_i = 2^{k_i - m_i}$ and $y_i = 2^{k_i}$ respectively where $k_i = \frac{m_i \cdot 2^{m_i}}{(2^{m_i}-1)}$

Where $m_i = i$ for all positive integers $i \geq 1$. QED.

(infinitely many)